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Quantum chaos in the Yang–Mills–Higgs system at finite temperature

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Abstract. Quantum chaos in the finite-temperature Yang–Mills–Higgs system is studied. The energy spectrum of a spatially homogeneous $SU(2)$ Yang–Mills–Higgs system is calculated within thermofield dynamics. Level statistics of the spectra is studied by plotting nearest-level spacing distribution histograms. It is found that finite-temperature effects lead to a strengthening of chaotic effects, i.e. a spectrum which has the Poissonian distribution at zero temperature has the Gaussian distribution at finite temperature.

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1 Introduction

Quantum chaos is a relatively new area in physics and has been the subject of extensive studies for the last two decades [1–6]. It has found applications in atomic and molecular physics, nuclear physics and condensed matter physics. In the past few years there is a growing interest in quantum chaos in particle physics, too. Being the quantum theory of classically chaotic systems the quantum chaology studies fluctuations in the energy spectra and wave functions of such systems. It is well known that the energy spectra of systems whose classical counterparts are chaotic have the same statistical properties as those for random matrices. Therefore one of the main topics in the quantum chaology is to study the statistical properties of the classically chaotic systems. Recently energy fluctuations and quantum chaos in hadrons and QCD have become a subject of theoretical studies [7–12]. In particular, it is found that the quark–gluon system in QCD is governed by quantum chaos in both confined and deconfined phases [8,9]. The statistical analysis of the measured meson and baryon spectra shows that the quantum chaos phenomenon occurs in these systems [10]. The study of the charmonium spectral statistics and its dependence on color screening has established the quantum chaotic behavior [7]. It was claimed that such a behavior could be the reason for J/Ψ suppression [7].

In recent years there has been considerable interest to determine the role of dynamical chaos in field theories [13–21]. Chaotic properties of Yang–Mills [13,15], Yang–Mills–Higgs [15–19] and Abelian Higgs [20] systems have been treated. The main point in these considerations is the fact that the Hamiltonians of the Yang–Mills and Yang–Mills–Higgs systems can be written in the same form as those for the coupled non-linear oscillators. This allows one to use for their treatment the same methods as in the case of coupled non-linear oscillators. Quantum chaos in Yang–Mills–Higgs system was also studied recently [22,23]. However, all works on chaos in field theories and hadrons are restricted to considering the zero-temperature cases.

In this paper we study quantum chaos at finite temperature in the Yang–Mills–Higgs system. Recent advances in heavy ion collision experiments allow one to create hot and dense hadronic and quark–gluon matters. The role of finite-temperature effects in such systems becomes important. Especially in quark–gluon or nuclear matter hadrons behave as complex systems where strong level fluctuations can be observed. Thus the role of finite-temperature effects as well as level fluctuations are crucial in such a systems, which obviously leads to a need for studying quantum chaos at finite temperature. In particular, finite-temperature effects cause fluctuations in their energy spectra.

We address the problem of the heat-bath in quantum chaos through the thermofield dynamics (TFD) formalism, a real time finite-temperature field theory [24–28]. TFD is interesting for our proposal here by its remarkable algebraic structure (this is not the case for the Matsubara [29] or the Schwinger–Keldysh [30] formalisms). Actually the central ideas of TFD involve an algebraic doubling in the degrees of freedom and a Bogolyubov trans-

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formation giving rise to the thermal variables. As we will show, this TFD prescription is a useful tool to explore finite-temperature effects in the energy fluctuations of the Yang–Mills–Higgs Hamiltonian, represented in terms of the annihilation and creation operators.

In Sect. 2 we present the Yang–Mills–Higgs system at zero temperature. In Sect. 3 the Yang–Mills–Higgs system at finite temperature is studied using the TFD formalism. In Sect. 4 we present numerical results for finite temperature and compare them to zero-temperature results, thus bringing out the crucial role of a finite temperature in the Yang–Mills–Higgs system and quantum chaos. Finally in Sect. 5 we present some conclusions and new directions to pursue the important question of quantum chaos.

2 Zero-temperature case

The Lagrangian for a Yang–Mills–Higgs system with $SU(2)$ symmetry is given by

$$L = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \frac{1}{2}(D_\mu\phi)^+(D^\mu\phi) - V(\phi), \quad (1)$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gA^b_\mu A^c_\nu,$$
$$(D_\mu\phi) = \partial_\mu\phi - \mathrm{i}gA^b_\mu T^b\phi,$$

with $T^b = \sigma/2$, $b = 1, 2, 3$ generators of the $SU(2)$ algebra, and g is a coupling constant. The potential of the scalar (Higgs) field is

$$V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4,$$

where μ and λ are constants. Here we give a brief description of the non-thermal case [22]. In $(2+1)$-dimensional Minkowski space and for spatially homogeneous Yang–Mills and Higgs fields which satisfy the conditions

$$\partial_i A^a_\mu = \partial_i\phi = 0, \qquad i = 1, 2,$$

and in the gauge $A^\alpha_0 = 0$, the Lagrangian can be written as

$$\begin{aligned} L &= \dot{\boldsymbol{\phi}}^2 + \frac{1}{2}(\dot{\boldsymbol{A}}_1^2 + \dot{\boldsymbol{A}}_1^2) \\ &- g\left[\frac{1}{2}\boldsymbol{A}_1^2\boldsymbol{A}_2^2 - \frac{1}{2}(\boldsymbol{A}_1\boldsymbol{A}_2)^2 + (\boldsymbol{A}_1^2 + \boldsymbol{A}_2^2)\boldsymbol{\phi}^2 \right. \\ &\left. - (\boldsymbol{A}_1\boldsymbol{\phi})^2 - (\boldsymbol{A}_2\boldsymbol{\phi})^2\right] - V(\boldsymbol{\phi}), \end{aligned} \quad (2)$$

where $\boldsymbol{\phi} = (\phi^1, \phi^2, \phi^3)$, $\boldsymbol{A}_1 = (A^1_1, A^2_1, A^3_1)$ and $\boldsymbol{A}_2 = (A^1_2, A^2_2, A^3_2)$.

To treat the chaotic dynamics it is convenient to use the canonical formalism and work with the Hamiltonian of the system instead of the Lagrangian. The Hamiltonian of the system can be written as [22]

$$H = \frac{1}{2}(p_1^2 + p_2^2) + g^2v^2(q_1^2 + q_2^2) + \frac{1}{2}g^2q_1^2q_2^2, \quad (3)$$

where $\boldsymbol{\phi}_0 = (0, 0, v)$ $q_1 = A^1_1$, $q_2 = A^2_2$ (the other components of the Yang–Mills fields are zero) $p_1 = \dot{q}_1$ and $p_2 = \dot{q}_2$, with $\omega^2 = 2g^2v^2$ being the mass term of the Yang–Mills field. This is the Hamiltonian of the classical system. Replacing p_i and q_i by operators and introducing the following creation and destruction operators:

$$\hat{a}_k = \sqrt{\frac{\omega}{2}}\hat{q}_k + \mathrm{i}\sqrt{\frac{1}{2\omega}}\hat{p}_k, \quad \hat{a}^+_k = \sqrt{\frac{\omega}{2}}\hat{q}_k - \mathrm{i}\sqrt{\frac{1}{2\omega}}\hat{p}_k,$$

we obtain the corresponding quantum Hamiltonian:

$$H = H_0 + \frac{1}{2}g^2V, \quad (4)$$

where

$$H_0 = \omega(a_1a_1^+ + a_2a_2^+ + 1),$$

and

$$V = \frac{1}{4\omega^2}(a_1 + a_1^+)^2(a_2 + a_2^+)^2,$$

with $\omega^2 = 2g^2v^2$, and the operators $\hat{a}_k$ and $\hat{a}^+_l$ satisfy the commutation relations $[\hat{a}_k, \hat{a}^+_l] = \delta_{kl}$, $k, l = 1, 2$. The eigenvalues of this Hamiltonian are calculated by numerical diagonalization of the truncated matrix of the quantum Yang–Mills–Higgs Hamiltonian in the basis of the harmonic oscillator wave functions [22]. The matrix elements of H_0 and V are

$$\langle n'_1, n'_2|H_0|n_1, n_2\rangle = \omega(n_1 + n_2 + 1)\delta_{n'_1n_1}\delta_{n'_2n_2},$$

and

$$\begin{aligned} &\langle n'_1, n'_2|V|n_1, n_2\rangle = \frac{1}{4\omega^2} \\ &\times\left\{\sqrt{n_1(n_1-1)}\delta_{n'_1n_1-2}\right. \\ &+\left.\sqrt{n_1(n_1-1)}\delta_{n'_1n_1+2} + (2n_1+1)\delta_{n'_1n_1}\right\} \\ &\times\left\{\sqrt{n_2(n_2-1)}\delta_{n'_2n_2-2}\right. \\ &+\left.\sqrt{n_2(n_2-1)}\delta_{n'_2n_2+2} + (2n_2+1)\delta_{n'_2n_2}\right\}. \end{aligned}$$

The numerical procedure for the diagonalization of this matrix is described by Salasnich [22]. We use the same method in the case of finite-temperature calculations.

3 Finite-temperature case

To treat quantum chaos in the finite-temperature Yang–Mills–Higgs system, we apply thermofield dynamics (TFD). TFD is a real time operator formalism of quantum field theory at finite temperature in which any physical system can be constructed from a temperature-dependent vacuum $|0(\beta)\rangle$ which is a pure state. The thermal average of any operator is equal to the expectation value between the pure vacuum state $|0(\beta)\rangle$, defined by applying

Bogolyubov transformations to the usual vacuum state. Furthermore, TFD has two main features. The first one is the doubling of the Fock space such that the original Fock space and its double are defined as non-tilde and tilde space respectively. All operators are also doubled, and the finite-temperature creation and annihilation operators are constructed by a Bogoluybov transformation between tilde and non-tilde operators. This is the same procedure as in writing down the vacuum state at finite temperature. Mathematically, the field operators have the following properties:

$$\begin{aligned} \widetilde{(A_i A_j)} &= \tilde{A}_i \tilde{A}_j, \\ \widetilde{(cA_i + A_j)} &= c^* \tilde{A}_i + \tilde{A}_j, \\ \widetilde{(A_i^*)} &= (\tilde{A}_i)^+, \\ \widetilde{(\tilde{A}_i)} &= A_i, \\ [\tilde{A}_i A_j] &= 0. \end{aligned}$$

The Yang–Mills–Higgs Hamiltonian in TFD is given as

$$\hat{H} = H - \tilde{H}, \tag{5}$$

where H is given by (4) and $\tilde{H}$ is given as

$$\tilde{H} = \tilde{H}_0 + \frac{1}{2} g^2 \tilde{V},$$

with

$$\tilde{H}_0 = \omega(\tilde{a}_1 \tilde{a}_1^+ + \tilde{a}_2 \tilde{a}_2^+ + 1),$$

and

$$\tilde{V} = \frac{1}{4\omega^2} (\tilde{a}_1 + \tilde{a}_1^+)^2 (\tilde{a}_2 + \tilde{a}_2^+)^2. \tag{6}$$

It has been established [31] that in an algebraic approach the doubled set of operators may be considered as a set of operators that relate to the physical observables, O, and a second set that are generators of symmetries, $\hat{O}$. The hat operators are responsible, in particular, for the time development and are needed for the purpose of scattering, decay and any transitions between states. The physical observables lead to the quantities that are measured in experiment. Both for physical observables and generators of symmetry after Bogolyubov transformations leading to finite-temperature creation and annihilation operators and to a pure vacuum state, only the non-tilde operators are required for getting the appropriate matrix elements. However, it is clear that for an analysis of any system at finite temperature both set of operators, O and $\hat{O}$, are needed since it is important to generate the appropriate symmetry of the system in any time development while considering the matrix element we have decided to investigate the chaotic behavior for the Hamiltonian that relates to the physical observables H and the Hamiltonian that generates the symmetry of the system, $\hat{H}$. Results of the finite-temperature quantum chaos in the Yang–Mills–Higgs system will be displayed for the physical observables, and for the generators of symmetry, $\hat{H}$. Only then we will draw conclusions about the approach of quantum chaotic behavior in the finite-temperature quantum field theory case of Yang–Mills–Higgs theory.

First we need to rewrite the non-tilde part of the Hamiltonian in the temperature-dependent form using the Bogolyubov transformations which are given by

$$\begin{aligned} a_k &= a_k(\beta)\cosh\theta + \tilde{a}_k^+(\beta)\sinh\theta, \\ a_k^+ &= a_k^+(\beta)\cosh\theta + \tilde{a}_k(\beta)\sinh\theta, \\ \tilde{a}_k &= a_k^+(\beta)\sinh\theta + \tilde{a}_k(\beta)\cosh\theta, \\ \tilde{a}_k^+ &= a_k(\beta)\sinh\theta + \tilde{a}_k^+(\beta)\cosh\theta, \end{aligned}$$

where

$$\beta = \frac{\omega}{k_{\mathrm{B}} T},$$

where the tilde and non-tilde creation and annihilation operators satisfy the following commutation relations:

$$[a_k(\beta), a_l^+(\beta)] = \delta_{kl}, \quad [\tilde{a}_k(\beta), \tilde{a}_l^+(\beta)] = \delta_{kl}.$$

We have $l, k = 1, 2$ and $\sinh^2\theta = (\mathrm{e}^\beta - 1)^{-1}$. All other commutation relations are zero.

Then the temperature-dependent forms of H_0 and $\tilde{H}_0$ are

$$\begin{aligned} H_0 &= \omega \left\{ (F_1 + F_2)\cosh^2\theta \right. \\ &\quad \left. + (L_1 + L_2)\sinh^2\theta + (S_1 + S_2)\cosh\theta\sinh\theta + 1 \right\}, \\ \tilde{H}_0 &= \omega \left\{ (F_1 + F_2)\sinh^2\theta \right. \\ &\quad \left. + (L_1 + L_2)\cosh^2\theta + (S_1 + S_2)\cosh\theta\sinh\theta + 1 \right\}, \end{aligned}$$

where

$$\begin{aligned} F_k &= a_k^+(\beta) a_k(\beta), \\ L_k &= \tilde{a}_k(\beta) \tilde{a}_k^+(\beta), \\ S_k &= a_k^+(\beta) \tilde{a}_k^+(\beta) + \tilde{a}_k^+(\beta) a_k(\beta). \end{aligned}$$

For V and $\tilde{V}$ we have

$$\begin{aligned} V &= \frac{1}{4\omega^2} \left\{ A_1\cosh^2\theta + B_1\cosh\theta\sinh\theta + C_1\sinh^2\theta \right\} \\ &\quad \times \left\{ A_2\cosh^2\theta + B_2\cosh\theta\sinh\theta + C_2\sinh^2\theta \right\}, \\ \tilde{V} &= \frac{1}{4\omega^2} \left\{ A_1\sinh^2\theta + B_1\cosh\theta\sinh\theta + C_1\cosh^2\theta \right\} \\ &\quad \times \left\{ A_2\sinh^2\theta + B_2\cosh\theta\sinh\theta + C_2\cosh^2\theta \right\}, \end{aligned}$$

where

$$\begin{aligned} A_k &= (a_k(\beta) + a_k^+(\beta))^2, \\ B_k &= (a_k(\beta) + a_k^+(\beta))(\tilde{a}_k^+(\beta) + \tilde{a}_k(\beta)) \\ &\quad + (\tilde{a}_k^+(\beta) + \tilde{a}_k(\beta))(a_k(\beta) + a_k^+(\beta)), \end{aligned} \tag{7}$$

and

$$C_k = (\tilde{a}_k^+(\beta) + \tilde{a}_k(\beta))^2.$$

In the first approach the energy eigenvalues of the thermal Yang–Mills–Higgs system can be calculated by diagonalization of the following matrix:

$$R = \langle n_1' n_2', \tilde{n}_1' \tilde{n}_2' | H_0 + \frac{1}{2} g^2 V | n_1 n_2, \tilde{n}_1 \tilde{n}_2 \rangle. \tag{8}$$

The elements of the matrix can be calculated explicitly:

$$\begin{aligned}&\langle n_1'n_2',\tilde{n}_1'\tilde{n}_2'|H_0|n_1n_2,\tilde{n}_1\tilde{n}_2\rangle\\&=\omega\left\{(n_1+n_2+1)(1+2\sinh^2\theta)\delta_{n_1'n_1}\delta_{n_2'n_2}\right.\\&+\left\{n_1\delta_{n_1'n_1-1}\delta_{n_2'n_2}+n_2\delta_{n_2'n_2-1}\delta_{n_1'n_1}\right.\\&+(n_1+1)\delta_{n_1'n_1+1}\delta_{n_2'n_2}\\&\left.\left.+\ (n_2+1)\delta_{n_2'n_2+1}\delta_{n_1'n_1})\right\}\cosh\theta\sinh\theta\right\},\end{aligned}$$

and for V

$$\begin{aligned}&\langle n_1'n_2',\tilde{n}_1'\tilde{n}_2'|V|n_1n_2,\tilde{n}_1\tilde{n}_2\rangle\\&=\frac{1}{4\omega^2}\langle n_1'n_2',\tilde{n}_1'\tilde{n}_2'|\left\{A_1A_2\cosh^4\theta+C_1C_2\sinh^4\theta\right.\\&+(A_1C_2+B_1B_2+C_1A_2)\cosh^2\theta\sinh^2\theta\\&+(A_1B_2+B_1A_2)\cosh^3\theta\sinh\theta\\&\left.+\ (B_1C_2+C_1B_2)\cosh\theta\sinh^3\theta\right\}|n_1n_2,\tilde{n}_1,\tilde{n}_2\rangle.\end{aligned}\qquad(9)$$

The matrix elements of A_k, B_k and C_k are given as

$$\begin{aligned}&\langle n_1'n_2',\tilde{n}_1'\tilde{n}_2'|A_k|n_1n_2,\tilde{n}_1\tilde{n}_2\rangle\\&=\sqrt{n_k(n_k-1)}\delta_{n_k'n_k-2}+(2n_k+1)\delta_{n_k'n_k}\\&+\sqrt{(n_k+1)(n_k+2)}\delta_{n_k'n_k+2},\\&\langle n_1'n_2',\tilde{n}_1'\tilde{n}_2'|B_k|n_1n_2,\tilde{n}_1\tilde{n}_2\rangle\\&=2n_k\delta_{n_k'n_k-1}+(2n_k+1)\delta_{n_k'n_k+1},\end{aligned}$$

and

$$\begin{aligned}&\langle n_1'n_2',\tilde{n}_1'\tilde{n}_2'|C_k|n_1n_2,\tilde{n}_1\tilde{n}_2\rangle\\&=\sqrt{n_k(n_k-1)}\delta_{n_k'n_k-2}+(2n_k+1)\delta_{n_k'n_k}\\&+\sqrt{(n_k+1)(n_k+2)}\delta_{n_k'n_k+2}.\end{aligned}$$

The calculation of the matrix element of $\hat{H}$, the generator of symmetry, gives us the following matrix:

$$\begin{aligned}Z&=\langle n_1'n_2',\tilde{n}_1'\tilde{n}_2'|H-\tilde{H}|n_1n_2,\tilde{n}_1\tilde{n}_2\rangle\\&=\omega\left\{-(2\cosh^2\theta)\delta_{n_1'n_1}\delta_{n_2'n_2}\right.\\&\left.+\ (n_1+n_2)\delta_{n_1'n_1}\delta_{n_2'n_2}\right\}\\&+\ \frac{g^2}{2\omega^2}\cosh^4\theta\left\{\sqrt{n_1(n_1-1)}\delta_{n_1'n_1-2}+(2n_1+1)\delta_{n_1'n_1}\right.\\&\left.+\ \sqrt{(n_1+1)(n_1+2)}\delta_{n_1'n_1+2}\right\}.\end{aligned}\qquad(10)$$

Diagonalizing the matrices R and Z numerically we obtain the energy eigenvalues of the Yang–Mills–Higgs system for the Hamiltonians H and $\hat{H}$ at finite temperature. As it was mentioned [19] the numerical energy levels depends on the dimension of the truncated matrix. We compute the numerical levels in double precision. The matrix dimension is 1156×1156, i.e. we calculate the first 1156 eigenvalues. Then the statistical properties of the spectra are found. We use standard unfolding procedure in order to remove the secular variation of the level density as a function of the energy E, for each value of the coupling constant the corresponding spectrum is mapped, by a numerical procedure described in [33].

One of the main characteristics of the statistical properties of the spectra is the level spacing distribution [2,3] function. In this work we calculate the nearest-neighbor level spacing distribution [2–4,7]. The nearest-neighbor level spacings are defined as $S_i = \tilde{E}_{i+1} - \tilde{E}_i$, where $\tilde{E}_i$ are the energies of the unfolded levels, which are obtained in the following way. The spectrum $\{E_i\}$ is separated into a smoothed average part and fluctuating parts. Then the number of the levels below E is counted and the following staircase function is defined:

$$N(E) = N_{\rm av}(E) + N_{\rm fluct}(E).$$

The unfolded spectrum is finally obtained with the mapping

$$\tilde{E}_i = N_{\rm av}(E_i).$$

Then the nearest level spacing distribution function $P(S)$ is defined as the probability of S lying within the infinitesimal interval $[S, S + {\rm d}S]$.

For the quantum systems which are chaotic in the classical limit this distribution function is the same as that of the random matrices [2,4]. For systems which are regular in the classical limit its behavior is close to a Poissonian distribution function. This distribution is usually taken to be a Gaussian with a parameter d:

$$P(H) \sim \exp\left(-{\rm Tr}\left\{HH^+\right\}/2d^2\right),$$

and the random matrix ensemble corresponding to this distribution is called the Gaussian ensemble. For Hamiltonians invariant under rotational and time-reversal transformations the corresponding ensemble of matrices is called the Gaussian orthogonal ensemble (GOE). It was established [2–4] that GOE describes the statistical fluctuation properties of a quantum system whose classical analog is completely chaotic. The nearest-neighbor level spacing distribution for GOE is described by the Wigner distribution:

$$P(S) = \frac{1}{2}\pi S\exp\left(-\frac{1}{4}\pi S^2\right). \qquad (11)$$

The usual way to study the level statistics is to compare the calculated nearest-neighbor level spacing distribution histogram with the Wigner distribution.

For systems whose classical motion is neither regular nor fully chaotic (mixed dynamics) the level spacing distribution will be intermediate between the Poisson and GOE limits. Several empirical functional forms for the distribution have been suggested for this case [4]. If the Hamiltonian is not time-reversal invariant, irrespective of its behavior under rotations, the Hamiltonian matrices are complex Hermitian and the corresponding ensemble is called a Gaussian unitary ensemble (GUE). If the Hamiltonian of the system is time-reversal invariant but not invariant under rotations, then the corresponding ensemble is called the Gaussian symplectic ensemble. In the next section we will study the numerical results for the level spacing and then analyze them to classify them in one of the these categories. It is to be emphasized that our Yang–Mills–Higgs Hamiltonian is time-reversal and rotational invariant.

4 Numerical results

On diagonalizing the vacuum expectation value of the Hamiltonian H, the physical observables, and $\hat{H}$, the generator of symmetry, given in (8) and (10), respectively the spectra are analyzed by considering the level spacing distribution. Then the criterion mentioned in the last section allows us to classify the system as chaotic or non-chaotic. In Figs. 1 and 2 we plot the level spacing distributions for different values of the parameters ω and θ for the energy spectrum calculated by diagonalizing of the matrix R. In Fig. 1 this distribution is given for the value $\omega = 0.92$ for which the non-thermal case level spacing distribution is chaotic [22]. As is seen from this figure for $\theta = 0$ it is the same as the results of non-thermal calculations [22]. By increasing the temperature it becomes closer to a Gaussian distribution that means strengthening of chaos in the thermal case. In Fig. 2 the level spacing distribution for $\omega = 0.01$ is plotted. For this value of ω the systems is regular at $\theta = 0$. However the increase of temperature leads to a chaotization of the system and $P(S)$ becomes closer to the Gaussian distribution. Figures 3 and 4 present level spacing distributions (for the same values of parameters as in Figs. 1 and 2) for the spectrum calculated by diagonalizing of the matrix Z. By comparing Figs. 1 and 2, and 3 and 4, it is clear that at zero temperature the level spacing distributions are the same for both methods. By increasing the temperature, the difference between the spacing distributions for different methods becomes considerable. It is clear from Figs. 1 and 2 that the level spacing distribution for $\omega = 0.92$, $\theta = 0.01$ is closer to the Gaussian distribution compared to the ones for $\omega = 0.01$, $\theta = 0.01$. The same behavior can be seen in histograms for $\omega = 0.92$, $\theta = 0.2$ and $\omega = 0.01$, $\theta = 0.2$. The reason for such behavior can be understood from Fig. 5 where the temperature T is plotted as a function of θ for various values of ω. It is clear that higher values of ω correspond to higher temperatures, while for smaller values of ω the temperature is also small.

However, for $\omega = 0.92$ the difference between the results for H and $\hat{H}$ is small while for $\omega = 0.01$ there is considerable difference in the results even for $\theta = 0.01$ (see Figs. 2 and 4). This indicates that the results from H, physical observables and $\hat{H}$, the generators of symmetry, are quite similar at high temperature while at low temperature the results are quite different. We know that at zero temperature H and $\hat{H}$ perform different functions. But the results indicate that at high temperature their chaotic behavior is similar.

Thus in both cases increasing the temperature leads to a smooth transition from a Poissonian to a Gaussian form in the level spacing distribution. Furthermore, at higher temperatures both H and $\hat{H}$ lead to quite similar results.

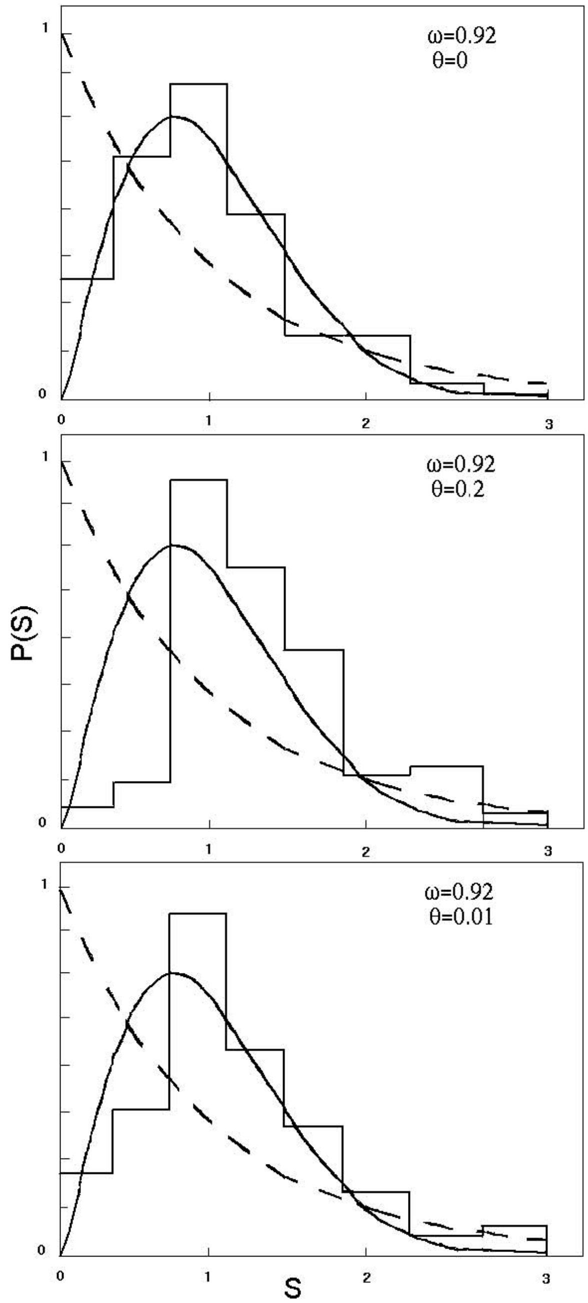

Fig. 1. The level spacing histograms for the Yang–Mills–Higgs system for the value of parameter $\omega = 0.92$

5 Conclusion

Summarizing, we have treated quantum chaos in gauge fields at finite temperature using a toy model, the $SU(2)$ Yang–Mills–Higgs system. To account for the finite-temperature effects we used the thermofield dynamics technique. The need for simultaneous exploration of the level fluctuations and the finite-temperature effects is dictated by recent advances in relativistic heavy ion collision experiments, that allow for the creation of hot and dense quark–gluon and hadronic matter [32].

The lattice QCD calculations of hadronic matter and quark–gluon matter indicate that both systems exhibit strong chaotic dynamics [8,11]. The present calculations in a toy model appear to support such conclusions. Furthermore the study of the Yang–Mills–Higgs system at

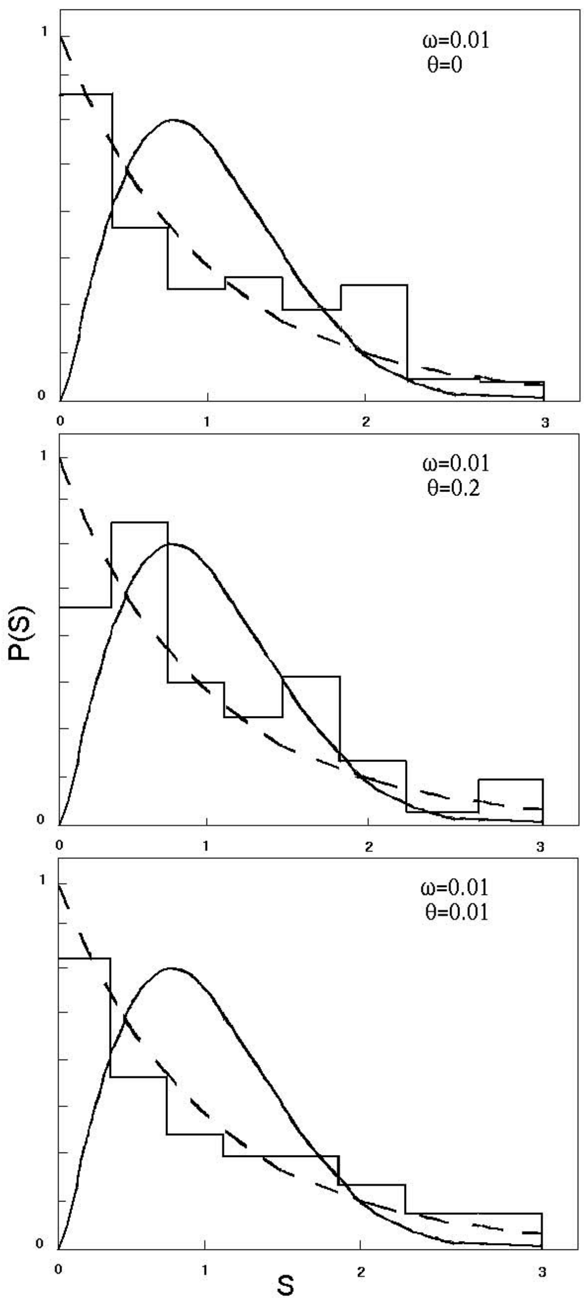

Fig. 2. The level spacing histograms for the Yang–Mills–Higgs system for the value of parameter $\omega = 0.01$

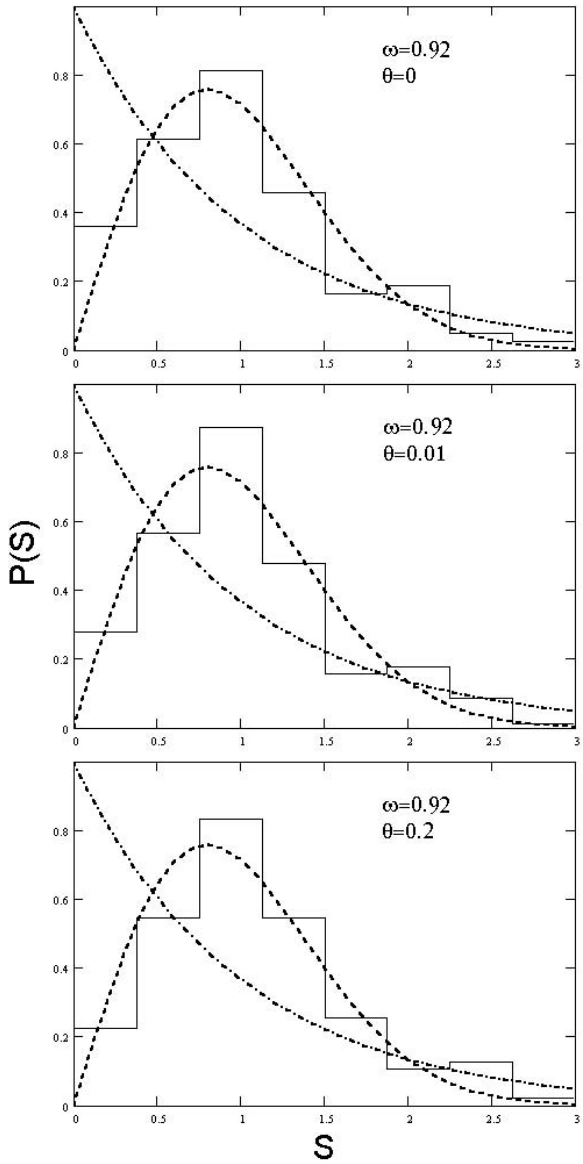

Fig. 3. The level spacing histograms for the Yang–Mills–Higgs system for the value of parameter $\omega = 0.92$

finite temperature establishes clearly that an increase of the temperature of the system strengthens level fluctuations in the spectra. Then a transition from the Poissonian to a Gaussian level spacing distribution does occur. It is to be anticipated that a study of the quark–gluon system in the relativistic heavy ion collisions at RHIC and later on at LHC would show the phenomenon of quantum chaos in a quantum field theoretic system. It is to be emphasized that a proper study in $(3+1)$ dimensions with full details of the Yang–Mills non-abelian field along with the Higgs scalar field is needed to make necessary conclusions for a quark–gluon plasma. However the present study provides an indication of the possible outcome in more realistic studies.

Finally it is interesting to summarise the results for Yang–Mills, Yang–Mills–Higgs and Yang–Mills–Higgs at finite temperature systems. It has been established that as the parameter, v, is increased, the role of Higgs is increased and this leads to a chaotic-to-regular system transition. A pure Yang–Mills system is chaotic in nature. The Higgs field leads to a less chaotic system. However the temperature leads to an increase in the random motion of particles. Thus it is anticipated that a Yang–Mills–Higgs system at finite temperature will provide a transition from a regular system to a more chaotic system. It is important to mention that in a quantum field theory the presence of Higgs, a scalar field, provides a spontaneous

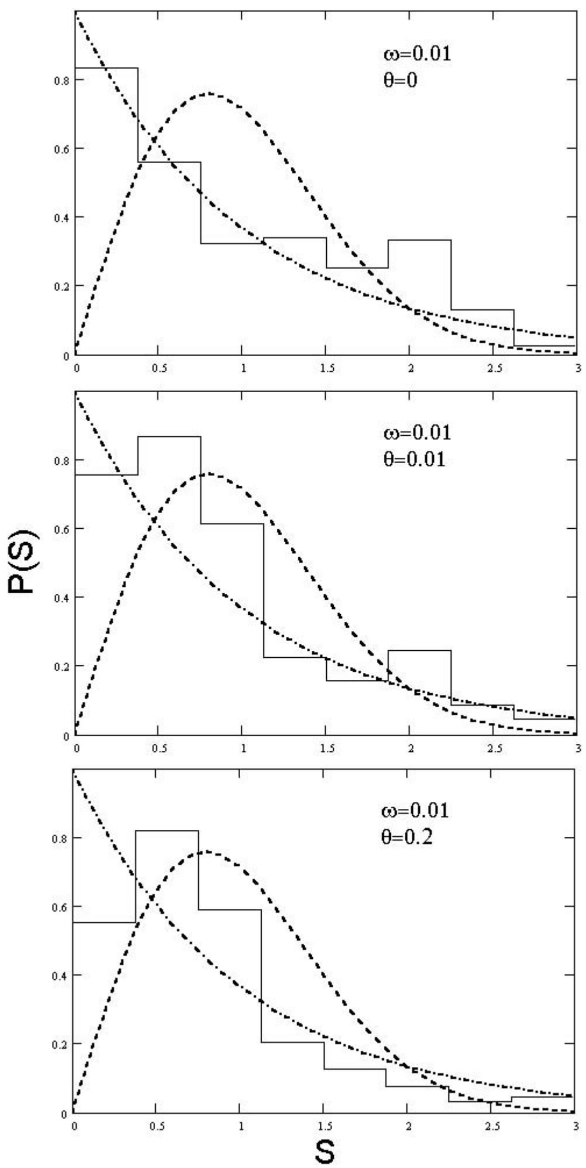

Fig. 4. The level spacing histograms for the Yang–Mills–Higgs system for the value of parameter $\omega = 0.01$

breakdown of the ground state (or vacuum) symmetry. The presence of temperature leads to a restoration of symmetry at some critical temperature. The results presented here, even though for a classical system, provide a reflection of the restoration of symmetry as the temperature is increased. Therefore for a classical system, Higgs leads the Yang–Mills system that is chaotic to a regular system while the temperature has an opposite effect leaving the system in a more chaotic state.

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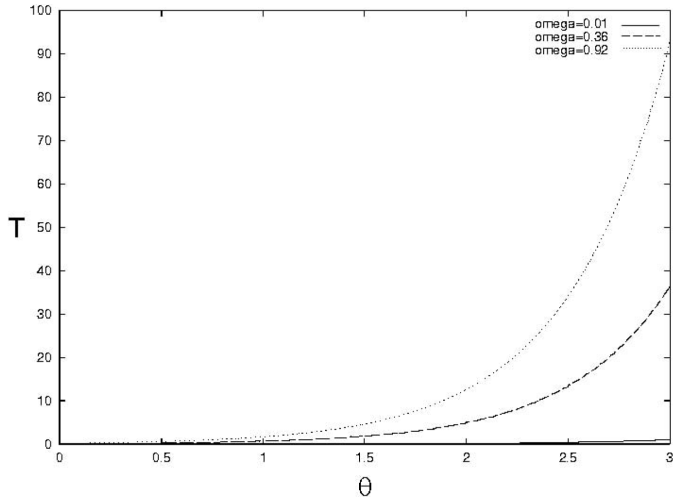

Fig. 5. Temperature as a function of θ

References

1. T.A. Brody et al., Rev. Mod. Phys. **53**, 358 (1981)
2. B. Eckhardt, Phys. Rep. **163**, 207 (1988)
3. M.C. Gutzwiller, Chaos in classical and quantum systems. (Springer, New York 1990)
4. T. Guhr, A. Mueller-Groeling, H.A. Weidenmueller, Phys. Rep. **299**, 189 (1998)
5. L.A. Bunimovich, Commun. Math. Phys. **65**, 259 (1979)
6. G.M. Zaslavsky, Stochastic behaviour of dynamical systems (Harwood, New York 1985)
7. Jian-zhong Gu et al., Phys. Rev. C **60**, 035211 (1999)
8. M.A. Halasz, J.J.M. Verbaarschot, Phys. Rev. Lett. **74**, 3920 (1995)
9. E. Bittner, H. Markum, R. Pullrich Quantum chaos in physical systems: from superconductors to quarks, hep-lat/0110222
10. V. Pascalusta, hep-ph/0201040
11. B.A. Berg et al., hep-lat/0007008
12. D. Bukta, G. Karl, B.G. Nickel, chao-dyn/9910026
13. G.K. Savvidy, Phys. Lett. B **130**, 303 (1983)
14. G.K. Savvidy, Nucl. Phys. B **246**, 302 (1984)
15. G.K. Savvidy, Phys. Lett. B **159**, 325 (1985)
16. T. Kawabe, S. Ohta, Phys. Lett. B **334**, 127 (1994)
17. T. Kawabe, S. Ohta, Phys. Rev. D **44**, 1274 (1991)
18. L. Salasnich, Phys. Rev. D **52**, 6189 (1995)
19. L. Salasnich, Mod. Phys. Lett. A **10**, 3119 (1995)
20. T. Kawabe, Phys. Lett. B **343**, 254 (1995)
21. S.G. Matinyan, B. Muller, Phys. Rev. Lett. **78**, 2515 (1997)
22. L. Salasnich, Mod. Phys. Lett. A **12**, 1473 (1997)
23. L. Salasnich, J. Math. Phys. **40**, 4429 (1999)
24. H. Umezawa, H. Matsumoto, M. Tachiki, Thermofield dynamics (North-Holland. Amsterdam 1982)
25. Y. Takahashi, H. Umezawa, Collective Phenomena **2**, 55 (1975) (reprinted in Int. J. Mod. Phys. A **10**, 1755 (1996))
26. A. Das, Finite temperature field theory (World Scientific, New York 1997)
27. Thermo Field Theories and Their Applications, edited by Y.X. Gui, F.C. Khanna, Z.B. Su (World Scientific, Singapore 1996)

28. A. Rakhimov, F.C. Khanna, Phys. Rev. C **64**, 064907 (2001)
29. T. Matsubara, Prog. Theor. Phys. **14**, 351 (1955)
30. J. Schwinger, J. Math. Phys. **2**, 407 (1961); L.V. Keldysh, Sov. Phys. JETP **20**, 1018 (1964)
31. A.E. Santana, F.C. Khanna, Phys. Lett. A **203**, 68 (1995)
32. Quark Matter 2001, Proceedings of the 15th International Conference on Ultra-Relativistic Heavy Ion Collisions, edited by T.J. Hallman, D.E. Kharzeev, J.T. Mitchell, T. Ullrich (North-Holland, Amasterdam 2002); Nucl. Phys. A **698**, 1 (2002)
33. V.R. Manfredi, Lett. Nuouo Cimento **40**, 135 (1984)